ENR355 Robotics and Sensors

Xiang Li

Spring 2026

COE COLLEGE

Current road map to ball balancer bot:

In theory:

- DOF of rigid body
- DOF of a bot
- Rigid body motions
- Forward kinematics
- PID controls
- Velocity kinematics and statistics
- Inverse kinematics

Hands-on:

- PCB soldering
- 2D graphic design and laser cutting
- 3D CAD design and 3D printing
- CNC machining

System integration:

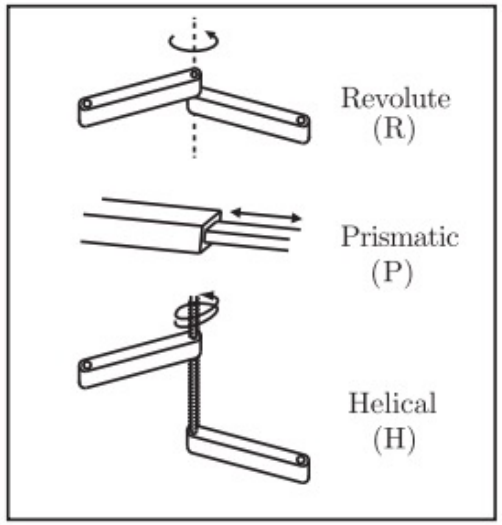
- Making joints and links
- Motor control (motion planning)
- Sensory link
- Sensor feedback control (calibration)
- Coding
- Simulation

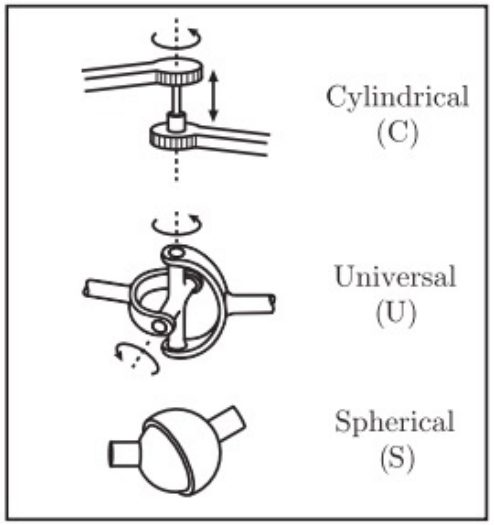
COE COLLEGE

DOF of a bot?

DOF of a free moving rigid body in 3D: 6

$$\text{degrees of freedom} = (\text{sum of freedoms of the bodies}) - (\text{number of independent constraints}). \quad (2.3)$$





Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

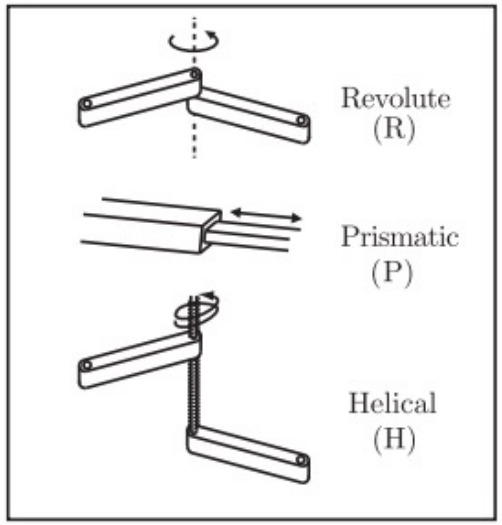
Figure 2.3: Typical robot joints.

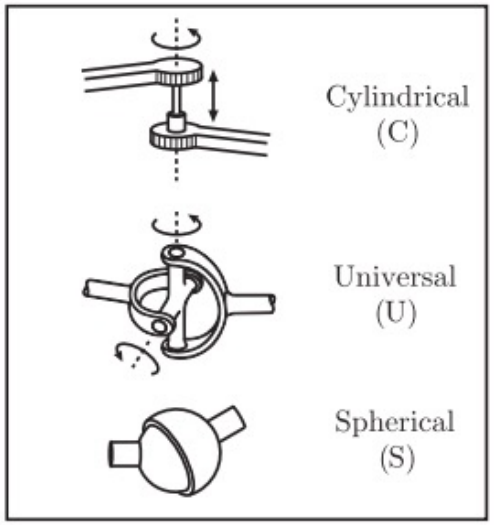
COE COLLEGE

DOF of a bot?

DOF of a free moving rigid body in 3D: 6

$$\text{degrees of freedom} = (\text{sum of freedoms of the bodies}) - (\text{number of independent constraints}). \quad (2.3)$$





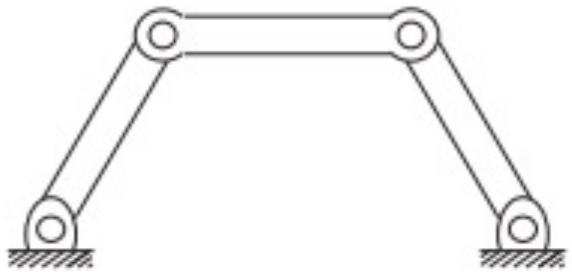
Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

Figure 2.3: Typical robot joints.

COE COLLEGE

Grübler's Formula

degrees of freedom = (sum of freedoms of the bodies) −
(number of independent constraints). (2.3)

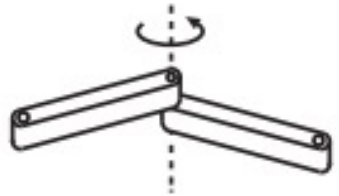


Proposition 2.2. *Consider a mechanism consisting of N links, where ground is also regarded as a link. Let J be the number of joints, m be the number of degrees of freedom of a rigid body ($m = 3$ for planar mechanisms and $m = 6$ for spatial mechanisms), f_i be the number of freedoms provided by joint i, and c_i be the number of constraints provided by joint i, where $f_i + c_i = m$ for all i. Then*

$$\text{dof} = \underbrace{m(N-1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^{J} c_i}_{\text{joint constraints}}$$

This formula holds only if all joint constraints are independent. If they are not independent then the formula provides a lower bound on the number of degrees of freedom.

Joint type	dof f	Constraints c between two planar rigid bodies
Revolute (R)	1	2
Prismatic (P)	1	2



Revolute (R)

Prismatic (P)

DOF= 3x(4-1)-(2+2+2+2)=1

Also:

$$\text{dof} = \underbrace{m(N-1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^{J} c_i}_{\text{joint constraints}}$$

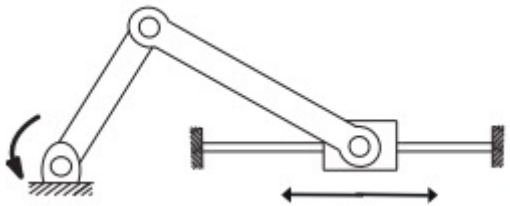
$$= m(N-1) - \sum_{i=1}^{J} (m - f_i)$$

$$= m(N-1-J) + \sum_{i=1}^{J} f_i.$$

COE COLLEGE

Grübler's Formula

$$\text{degrees of freedom} = (\text{sum of freedoms of the bodies}) - (\text{number of independent constraints}). \qquad (2.3)$$



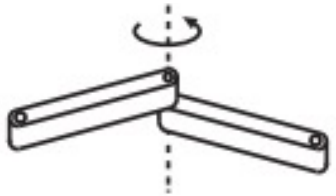
Proposition 2.2. *Consider a mechanism consisting of N links, where ground is also regarded as a link. Let J be the number of joints, m be the number of degrees of freedom of a rigid body ($m = 3$ for planar mechanisms and $m = 6$ for spatial mechanisms), f_i be the number of freedoms provided by joint i, and c_i be the number of constraints provided by joint i, where $f_i + c_i = m$ for all i. Then*

$$\text{dof} = \underbrace{m(N-1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^{J} c_i}_{\text{joint constraints}}$$

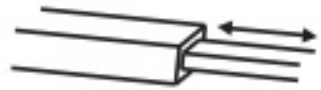
This formula holds only if all joint constraints are independent. If they are not independent then the formula provides a lower bound on the number of degrees of freedom.

DOF= ?

Joint type	dof f	Constraints c between two planar rigid bodies
Revolute (R)	1	2
Prismatic (P)	1	2



Revolute (R)



Prismatic (P)

Also:

$$\text{dof} = \underbrace{m(N-1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^{J} c_i}_{\text{joint constraints}}$$

$$= m(N-1) - \sum_{i=1}^{J} (m - f_i)$$

$$= m(N-1-J) + \sum_{i=1}^{J} f_i.$$

COE COLLEGE

Grübler's Formula is working when it's working

$$\text{degrees of freedom} = (\text{sum of freedoms of the bodies}) - (\text{number of independent constraints}). \quad (2.3)$$

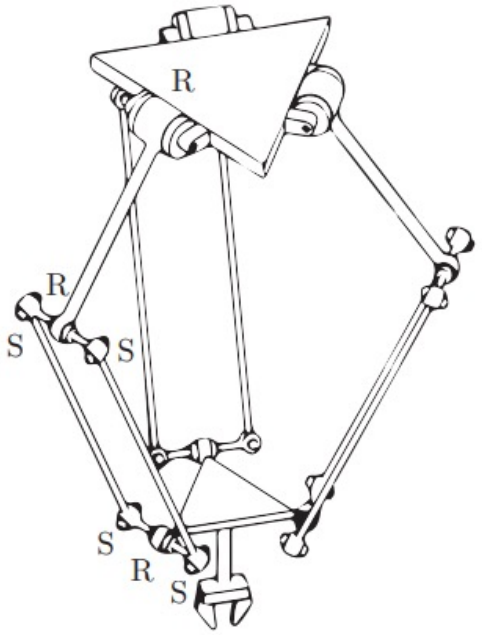


Figure 2.8: The Delta robot.

Example 2.7 (Delta robot)**.** The Delta robot of Figure 2.8 consists of two platforms – the lower one mobile, the upper one stationary – connected by three legs. Each leg contains a parallelogram closed chain and consists of three revolute joints, four spherical joints, and five links. Adding the two platforms, there are $N = 17$ links and $J = 21$ joints (nine revolute and 12 spherical). By Grübler's formula,

$$\text{dof} = 6(17 - 1 - 21) + 9(1) + 12(3) = 15.$$

Actual DOF = 3

Redundant links don't count!

COE COLLEGE