ENR145 Computational Methods:
Little follow-up on COMSOL multi-physics

Xiang Li

Spring 2026

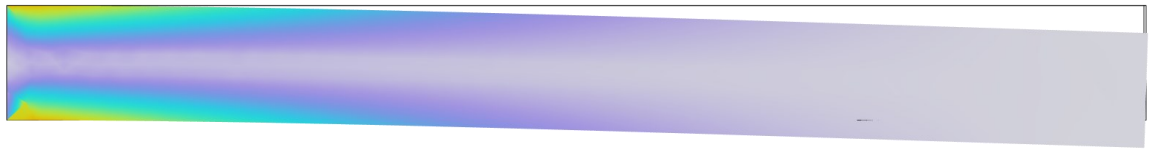
COE COLLEGE

What we did last time:
Multi-physics modeling of a wax beam

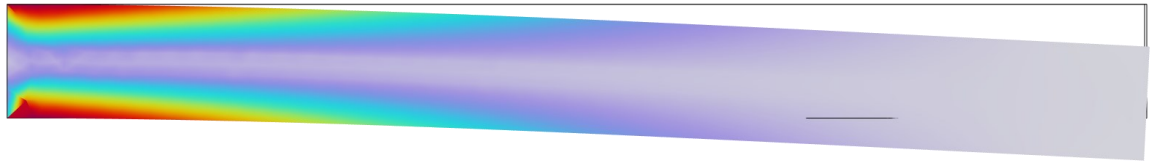
When E is a constant:

0h

0.1h



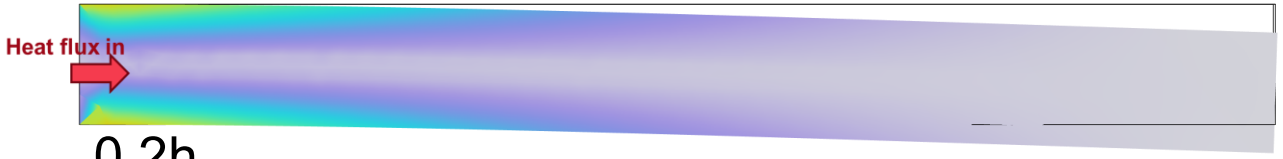
Max at 0.2h



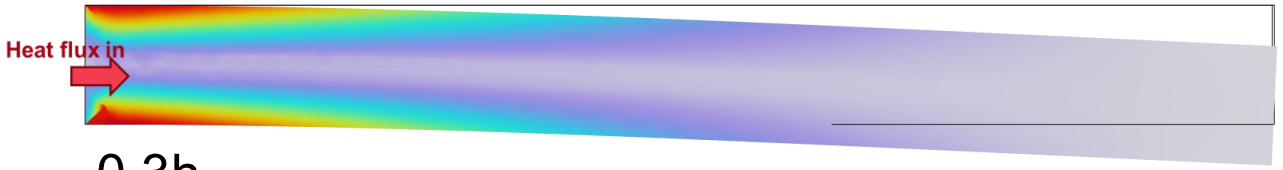
When E is temperature dependent:

0h

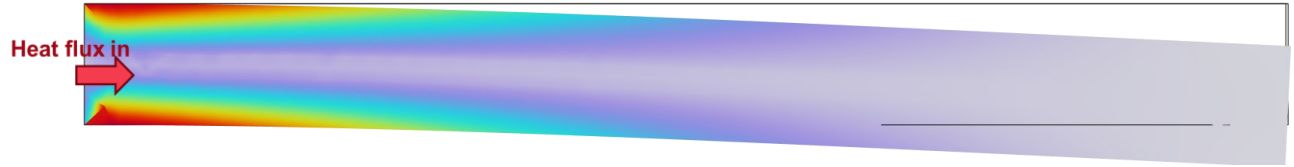
0.1h



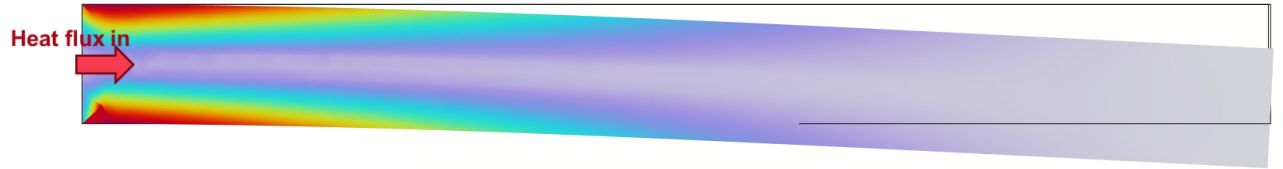
0.2h



0.3h



Max at 0.7h

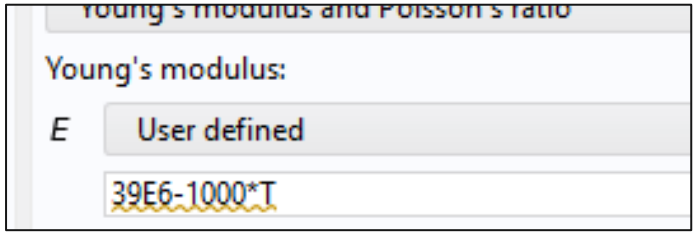


Heat transfer couple with solid mechanics

- Heat Transfer in Solids *(ht)*
  - Solid 1
  - Initial Values 1
  - Thermal Insulation 1
  - Heat Flux 1
  - Heat Flux 2
- Solid Mechanics *(solid)*
  - Linear Elastic Material 1
  - Free 1
  - Initial Values 1
  - Fixed Constraint 1
  - Body Load 1

Temperature gradient (K)

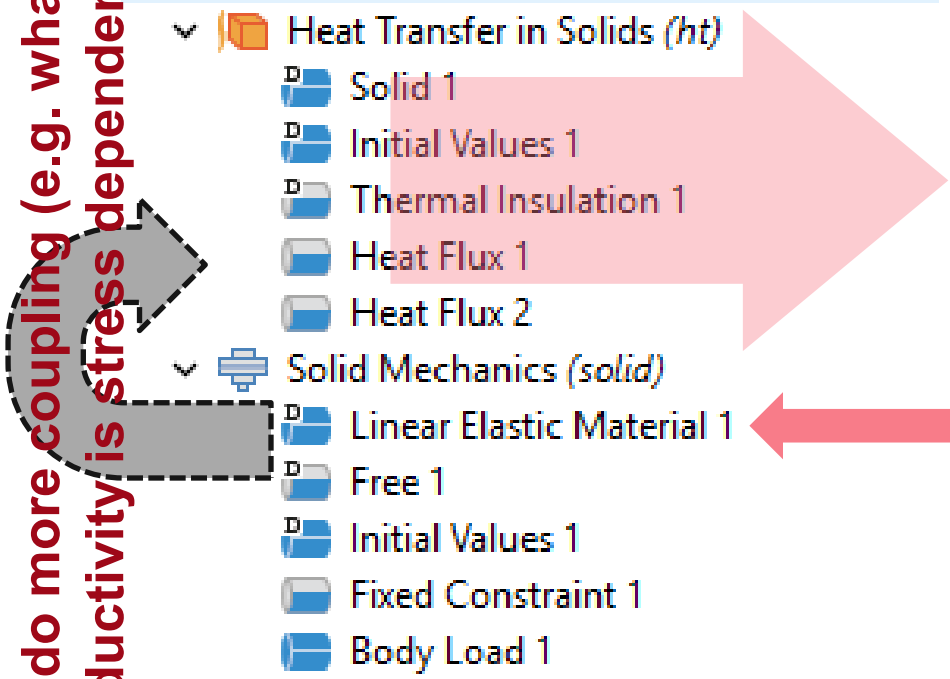
What is the meaning of this?



COE COLLEGE

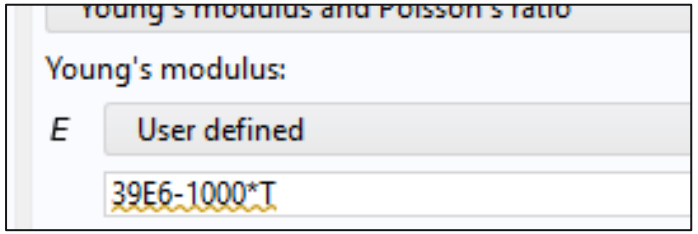
Solid mechanics can couple back, too

We can even do more coupling (e.g. what if thermo-conductivity is stress dependent)



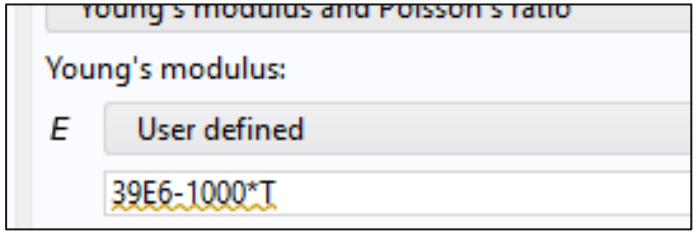
Temperature gradient (K)

What is the meaning of this?



COE COLLEGE

And the stiffness change might not be linear:



How do you "math" this?

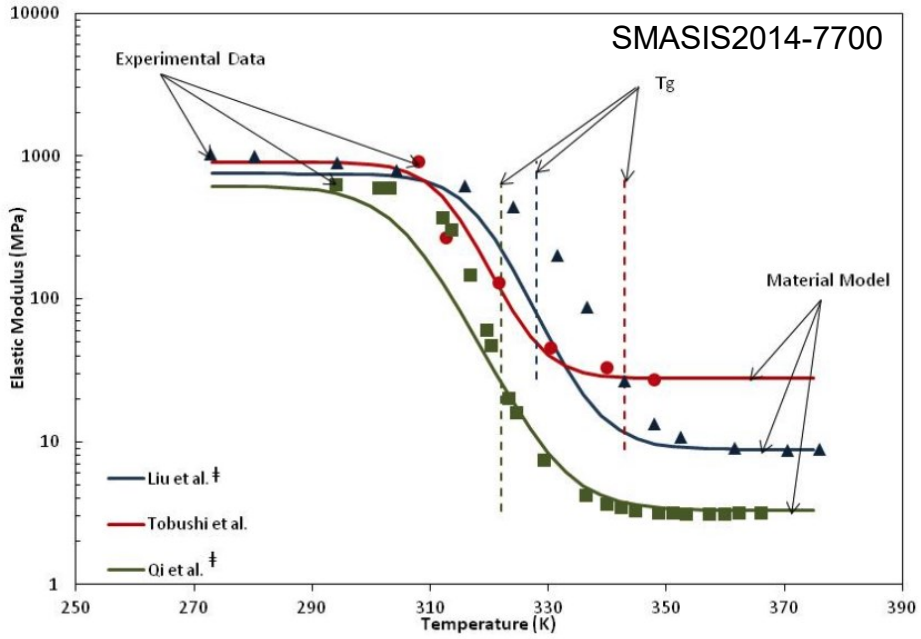
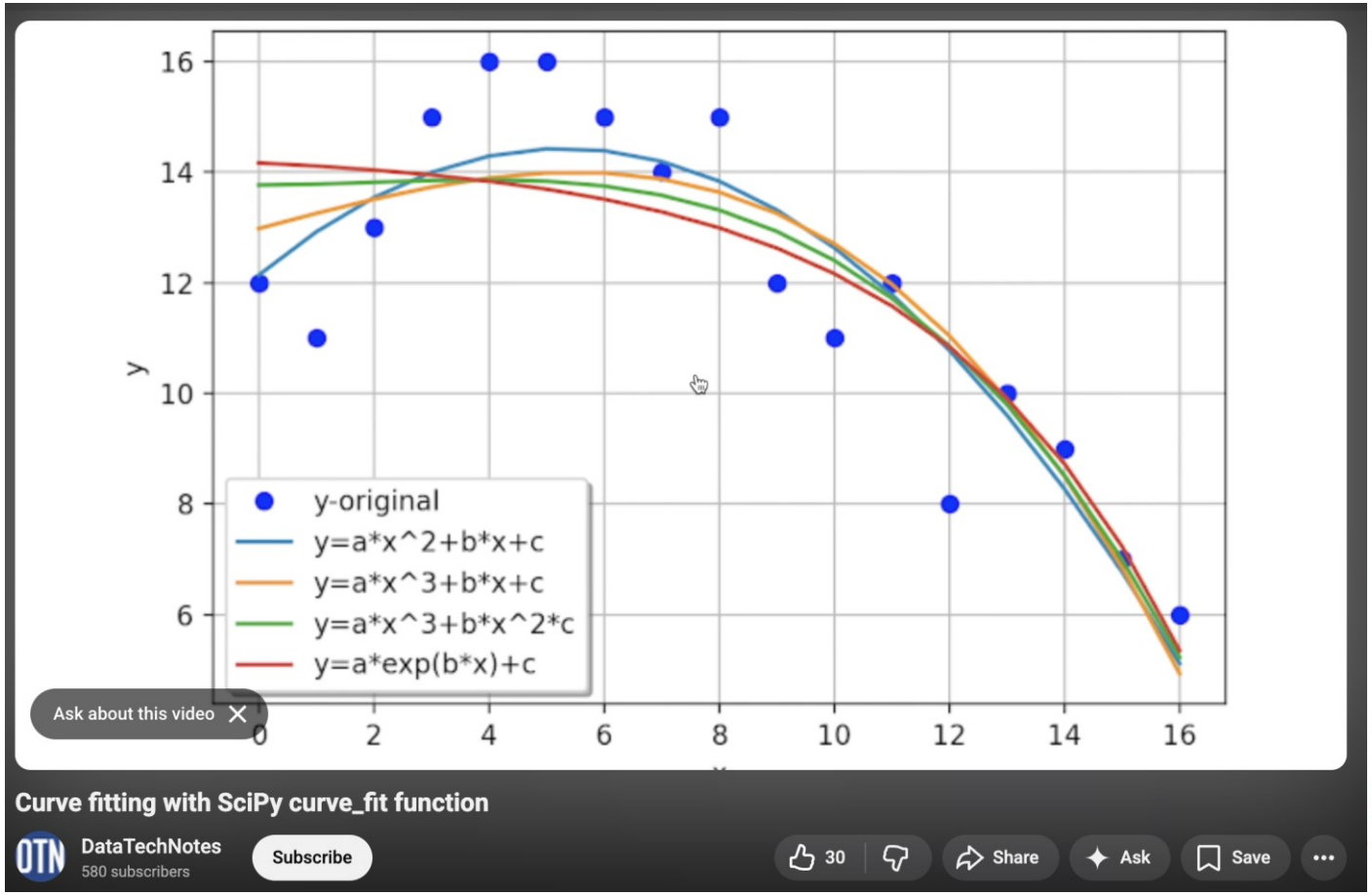


Figure 2. Elastic Modulus variation with Temperature – Model compared to experimental data.

Introducing: curve fitting

https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html



COLLEGE

S-shaped curve fitting speed run:

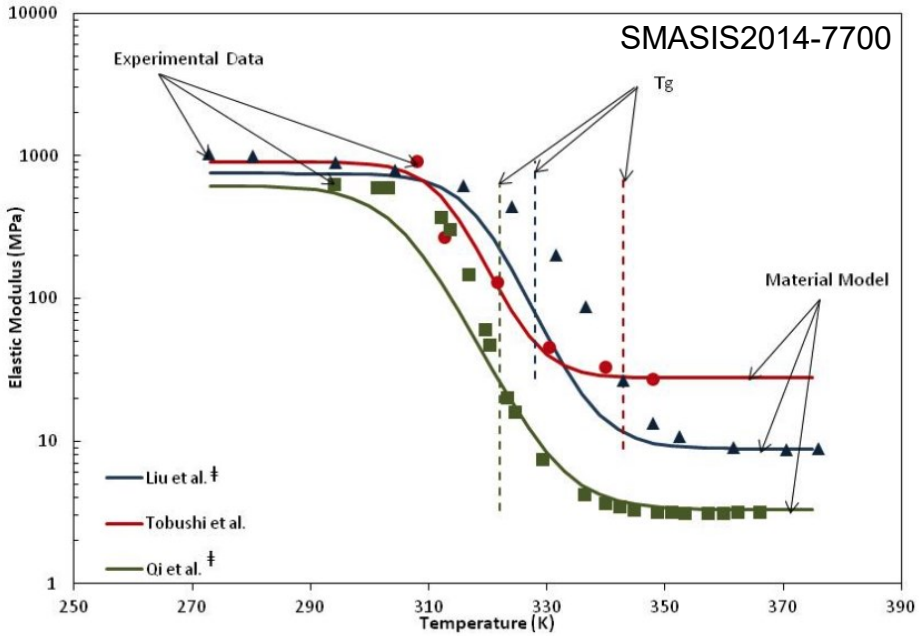
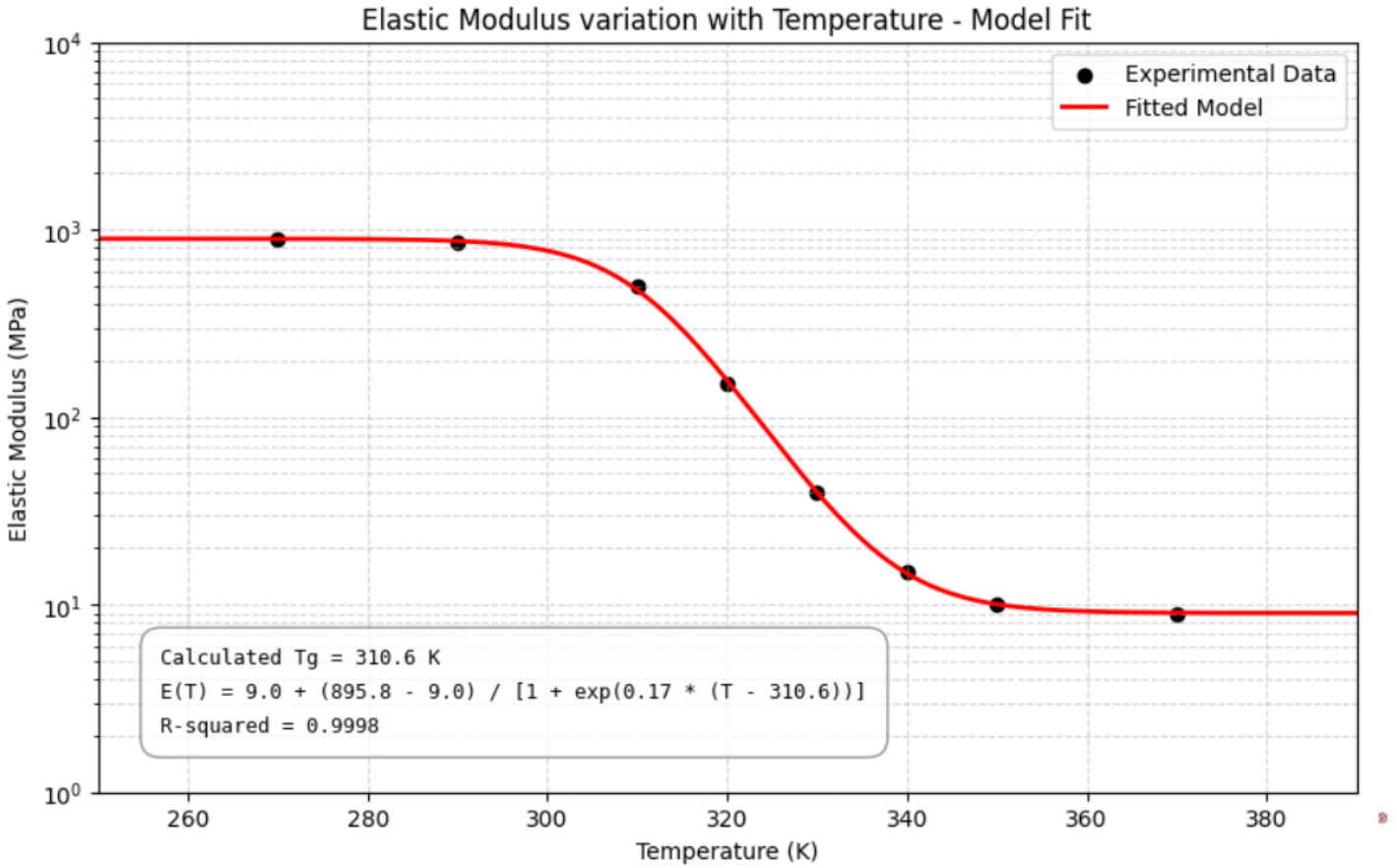


Figure 2. Elastic Modulus variation with Temperature – Model compared to experimental data.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from sklearn.metrics import r2_score
# run this box to import the library
```

The S-shaped curve can be fit with this equation:

$$E(T) = E_{\text{phase 1}} + \frac{E_{\text{phase 1}} - E_{\text{phase 2}}}{1 + e^{k(T - T_g)}}$$



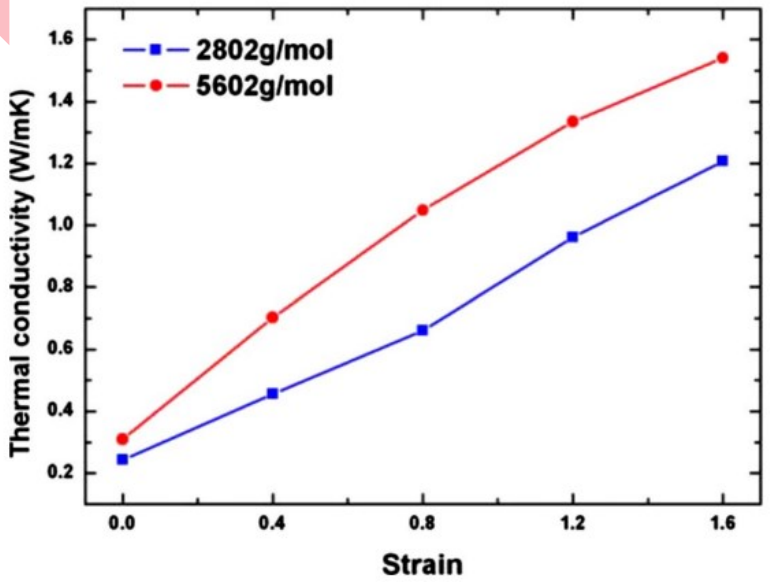
Now try this:

You have to play with the range to speed up the simulation.

```
E(T) = 9.0 + (895.8 - 9.0) / [1 + exp(0.17 * (T - 310.6))]
```

- Heat Transfer in Solids *(ht)*
  - Solid 1
  - Initial Values 1
  - Thermal Insulation 1
  - Heat Flux 1
  - Heat Flux 2

- Solid Mechanics *(solid)*
  - Linear Elastic Material 1
  - Free 1
  - Initial Values 1
  - Fixed Constraint 1
  - Body Load 1



COE COLLEGE